The material on organizational issues has been gathered from many industry presentations and conversations with industry RM practitioners.

## APPENDIX 11.A: Normal and Inverse Normal Approximations

Most spreadsheet programs and mathematical libraries have functions for the normal distribution and its inverse. The formulas in this appendix are useful for anyone programming a RM system (say, an EMSR algorithm). There are a number of such approximations circulating as folklore in the scientific programming community. The normal approximation we present is due from Abramowitz and Stegun [2] and Bagby [24] and is accurate up to four digits. The inverse normal distribution is based on Halley's method ([455]) and presented here as implemented by Acklam [3].

It is common in quantity-based RM practice to assume that the aggregate number of customers follows a normal distribution truncated to the left at zero. As is well-known, there are no closed-form expressions for the normal distribution and its inverse. In simulations (as well as in RM optimization algorithms such as the EMSRb) rational approximation functions to the normal distribution are used, which we describe next. Both are highly accurate and sufficient for most practical applications in RM.

The following approximation is accurate up to four digits.

**Approximation formula for $\tilde{F}(x) \approx P(X \leq x)$, where $X \sim$ Normal(0,1):**

**if** $x \geq 4.0$, $\tilde{F}(x) = 1.0$ **stop**
**if** $x \leq -4.0$, $\tilde{F}(x) = 0.0$ **stop**
**let**

$a_1 = 0.319381530$
$a_2 = -0.356563782$
$a_3 = 1.781477937$
$a_4 = -1.821255978$
$a_5 = 1.330274429$
$b = 0.2316419$
$c = \frac{1}{\sqrt{2\pi}} \approx 0.398942208$

**if** $x > 0.0$, **let**

$y = \frac{1}{1+bx}$
$\tilde{F}(x) = 1 - ((e^{-0.5x^2}c)(y(a_1 + y(a_2 + y(a_3 + y(a_4 + a_5 y))))))$

**else**

$y = \frac{1}{1-bx}$
$\tilde{F}(x) = (e^{-0.5x^2}c)(y(a_1 + y(a_2 + y(a_3 + y(a_4 + a_5 y)))))$

**end if**

The next approximation method for inverse normal is considered accurate to nine digits.

**Approximation formula for $\tilde{F}^{-1}(p) \approx F^{-1}(p) : (0,1) \to \Re$, and $F^{-1}(p) = x$ if and only if $P(X \leq x) = p$, where $X \sim$ Normal(0,1):**

**let**

$a_1 = -3.969683028665376E + 01$